

Strategy Impact: Trade-Size Formulas, Part 2

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Summary

Generally speaking, there are two principal requirements that a strategy must meet before it can be traded using real capital. First, the strategy must demonstrate positive risk-adjusted expectancy. Second, it must have effective money-management controls. In addressing the latter point in an earlier *Analysis Concepts* paper, “*Strategy Impact: Trade-Size Formulas*,” we explored four generally recognized trade-size techniques. In this paper, the Kelly Criterion and Fixed Ratio concepts will be introduced to the discussion. The inner workings of these approaches and the pros and cons of each method will be examined, while also assessing their effects on strategy performance-related attributes, such as the relationship between minimal risk and maximal return.

Introduction

Beat the Dealer, originally written in 1962 by Edward O. Thorp, introduced “The Kelly Gambling System,” also known as the Kelly Criterion¹. Considered by many as the originator of card counting because of his work in probability theory and achievements at the blackjack table, Thorp was also a mathematics professor and a very successful hedge-fund manager. His book leads one to consider money management and the optimal trade size used in

trading systems. A connection can be made between optimal trade size and the “Kelly bet.”

As noted above, developing a viable trading strategy requires that two fundamental objectives be satisfied. First, a trading strategy must exhibit positive risk-adjusted expectancy; second, its money-management technique must be effective. Expectancy is the probability of profitability based on historical data. The formula for expectancy is:

$$\text{Expectancy} = \frac{(\text{Probability of Winning Trade} * \text{Avg Win } \$) - (\text{Probability of Losing Trade} * \text{Avg Loss } \$)}$$

Money management, however, tends to be an overlooked aspect of trading. What is money management? Money-management methods are intended to address the question of how much capital should be risked on any one trade. It allows us to determine the optimal trade size for maximizing the long-term growth rate of capital while considering factors of risk. Typically, the risk component is either directly or indirectly incorporated in the trade-size formula, which produces the optimal trade size. Investing too little capital in each trade represents a missed opportunity when the trade is profitable. If the trade size is too large, one losing trade could cause a very large loss.

This leads the strategy trader to a dilemma: the trade-off between risk and return. The larger the trade size as a percentage of capital, the greater

the risk. It is also commonplace for traders or investors to determine their utility preference function through Monte Carlo testing or historical back-testing simulations. A utility function describes how the investor or trader feels toward varying degrees of risk and return. Monte Carlo simulations can be conducted to see these effects on a strategy's equity curve. Although historical drawdowns are not always representative of future risk, a greater sense of confidence may be achieved by using these techniques to observe the historical risk-and-return trade-off.

Strategy Model: Relative Strength Index (RSI)

The .eld and .tsw files that hold the trade-size formulas reviewed in this paper are available for download. Both trade-size methods were tested using the E-mini S&P 500 continuous contract, daytime only (@ES.D). As in the first paper, "Strategy Impact: Trade-Size Formulas," the strategy model used to program the trade-size formulas was the Relative Strength Index (RSI). Table 1 shows the buy and sell rules for this strategy.

Table 1: RSI Strategy Rules, RSI Length = 2

	Indicator	Action	Input Value
Buy Rule	RSI(2)	Crosses Below	30
Sell Rule	RSI(2)	Crosses Above	70

Kelly Criterion

The Kelly formula became publicized in a paper entitled, "A New Interpretation of the Information Rate," by John Larry Kelly, Jr., published in a 1956 edition of the Bell System Technical Journal². Around this time, a group of Bell Labs systems engineers in Texas were thinking about how to tackle a data communication problem, as the transmission of data over phone lines was subject to random levels of noise interference, especially at longer distances. As it turns out, determining optimal trade size is similar to the

problem related to data transmission over long-distance telephone lines. The objective of the Kelly Criterion is to maximize the long-term geometric wealth (defined as the compounded return from the re-investment of net profits) of a trading strategy, which is dependent on the sequence of winning and losing trades. The formula below defines a fixed fraction of capital to invest in each trade and should be based on the expectation (probability) of long-term capital growth. The formula for this method is:

$$TradeSize = W\% - \frac{1 - W\%}{Profit\ Factor}$$

Where:

TradeSize	=	the percentage of capital allocated to a trade
W%	=	the percentage of total trades that are winning trades (Percent Profitable in the Strategy Performance Report)
Profit Factor	=	the gross profit divided by gross loss. A value greater than 1.0 means the strategy has a positive net profit. (This field also appears in the Strategy Performance Report.)

As is the case for any position-size formula, the strategy must have positive risk-adjusted expectancy for any money management to be additive. In the case of the Kelly Criterion, a strategy that has a positive expectancy will maximize the geometric growth in returns through the re-investment of profits or trade-to-trade compounding of returns. In the Kelly formula above, profit factor and winning percentage of trades are the key ingredients in the balance of risk and return. As Table 2 illustrates, examples 1, 2 and 3 all have a constant profit factor of 1. As the percentage of winning trades increases or is greater than 50%, trade size will increase. Examples 4, 5 and 6 illustrate that with constant odds of winning at 60%, the trade size increases as the profit factor increases.

Table 2: Kelly Criterion Examples

Example 1			
W%	=		50%
Profit Factor	=		1.00
TradeSize	=		0%

Example 4			
W%	=		60%
Profit Factor	=		1.50
TradeSize	=		33%

Example 2			
W%	=		60%
Profit Factor	=		1.00
TradeSize	=		20%

Example 5			
W%	=		60%
Profit Factor	=		2.00
TradeSize	=		40%

Example 3			
W%	=		70%
Profit Factor	=		1.00
TradeSize	=		40%

Example 6			
W%	=		60%
Profit Factor	=		3.00
TradeSize	=		47%

There are, however, several issues with full Kelly trade sizes. For example, when winning trades outnumber losing trades, account size will increase. At the same time, the Kelly trade size gets larger and larger. In example 6, at 60% winners to losers and a 3:1 profit factor, trade size will be around 47% of current capital. Now, envision a scenario in which the next trade is not a winner and a severe drawdown will occur. This is an example of asymmetric leverage. It can occur when a strategy goes through a period of drawdowns. A simple formula to illustrate this concept is:

$$\frac{1}{(1 - MDD)} - 1$$

As an example, if a strategy experiences a 25% drawdown (MDD), a gain of 33% is needed for the equity to recover that drawdown. The formula above may be used to make this calculation.

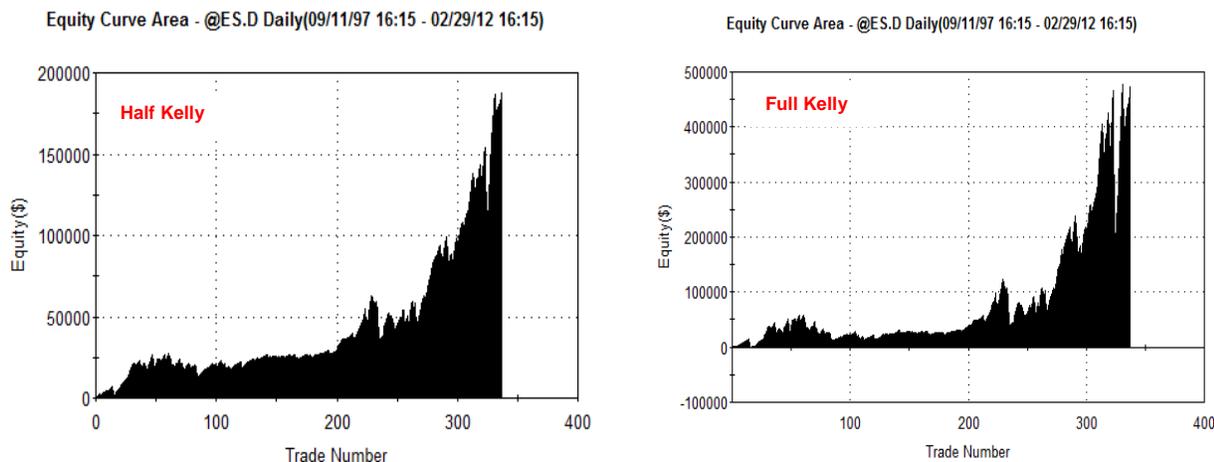
Strategies that employ anti-martingale money management (increasing position size after winning trades and decreasing position size after losing trades) typically have to address the issue of asymmetric leverage after experiencing a period of drawdowns because of the greatly reduced position size at the bottom of the drawdown. The Kelly Criterion is no different and is subject to large levels of drawdowns as position sizes increase. It is often said that some common sense should be applied in order to control levels of drawdown. This is the point when traders have to determine their utility function preference between risk and return. For

this reason, it is common for traders to use less than the full Kelly percentage. For example, if Kelly is recommending a 10% allocation, one might use half Kelly or 5%.

Kelly Criterion Strategy Performance

The RSI strategy was tested with the Kelly Criterion using the full Kelly and half Kelly. Daily data for the E-mini S&P 500 (@ES.D) was used, with a start date of 9/11/1997; initial capital was \$10,000. Applying the Kelly Criterion to futures data requires an initial margin value amount, which is currently \$5,000 when trading the E-mini contract. Figure 1 demonstrates the equity curves trading half Kelly and full Kelly.

Figure 1: Half Kelly equity curve (left), full Kelly equity curve (right)



Half Kelly, on the left side, has a more linear equity curve. Also, the average number of days it takes to make a new equity high is fewer. The reason for this difference is that at the time this strategy experiences its first periods of drawdowns, around the beginning of the 2000 bear market, full Kelly is trading more contracts than half Kelly. This results in dollar drawdowns that are greater. Beyond trade 100, expectancy begins to turn more positive and around trade 150 half Kelly is making new highs, while full Kelly does not make new highs in its equity until after trade 200. At this time, full Kelly is beginning to ramp up the number of contracts it trades, because it has built its equity back up, as has half Kelly, except that half Kelly can buy contracts equal to or more than it did prior to its equity peak.

A similar experience occurs around trade 325. Half Kelly experiences its drawdown, but then continues to make a new equity high, while full Kelly has yet to make a new high, as the strategy closes in on trade 350. Also, notice where the growth in the equity curve occurs. It begins around trade 250. This is the result of a combination of positive expectancy with optimal trade size. Geometric return becomes greatest during this period in the equity curve. Clearly, the sequence of trades is important to the money-management scheme. In the case of the Kelly Criterion, a sequence of trades that yields positive levels of expectancy, followed by periods of negative expectancy will experience heavy drawdowns with fully Kelly and most other anti-martingale methods.

Figure 2: Strategy Performance Report Statistics (Half Kelly)

TradeStation Performance Summary		
	All Trades	Long Trades
Total Net Profit	\$187,975.00	\$187,975.00
Gross Profit	\$534,100.00	\$534,100.00
Gross Loss	(\$346,125.00)	(\$346,125.00)
Profit Factor	1.54	1.54

Figure 3: Strategy Performance Report Statistics (Half Kelly)

Return Retracement Ratio	0.14
RJINA Index	70.74
Sharpe Ratio	0.14
K-Ratio	1.42

Strategy performance statistics using half Kelly (Figure 2) showed a greater profit ratio than full Kelly, at 1.54 versus 1.35. Half Kelly also produced a 1.42 K-Ratio and a 70.74 Rina Index (Figure 3), while full Kelly had a .89 K-Ratio and 17.42 Rina Index.

Fixed Ratio

In 1999, Ryan Jones wrote about the Fixed Ratio method in his book, *The Trading Game*³. The Fixed Ratio method could be classified as an anti-martingale position size approach. According to Jones, his method addresses the matter of balancing geometric growth versus risk or, more specifically, asymmetric leverage. The formula for this method is:

$$\text{TradeSize} = 0.5 * (1 + \sqrt{(1 + 8 * \frac{\text{NetProfit}}{\text{Delta}})})$$

In its basic form, the Fixed Ratio method is derived from the relationship between net profit accrued and the number of contracts traded, which Jones says is a fixed relationship. That is, as profits increase above a specified profit boundary, the number of contracts traded will increase by one. On the flip side, as the specified loss boundary (the same value as the specified profit level) is exceeded, the number of contracts traded will decrease by one. The designated profit and loss boundary is what Jones calls "delta." It is in the denominator, under net profit in the equation above.

As an example, assume delta is \$10,000. A strategy with profits of \$10,000 will add another contract. A higher delta value may translate into a slower rate of account growth, assuming profitable trading, especially for a smaller trading account. The higher delta means fewer contracts per the required size of the account. That is, it will take longer to make money because the hurdle that has to be overcome in terms of profits is greater. A higher delta as the designated profit boundary implies a more conservative approach to increasing trade size. A lower delta value allows for a quicker ramping up in contracts as the profit boundary is lower. This could be ideal for smaller accounts if an earlier series of trades exhibited positive risk-adjusted expectancy and that positive expectancy continues.

The delta value is a subjective value that the trader must choose, and there is no magic number. Many suggest using the historically back-tested maximum drawdown per contract, average drawdown per contract, or the maximum losing trade. Delta could also be the margin requirement on one futures contract (i.e. E-mini S&P 500 contract is \$5,000 as of February 29, 2012).

There are two key attributes of the Fixed Ratio method. First, considering that the strategy has a positive risk-adjusted expectancy, it does not take as long to ramp up the number of contracts traded early on compared to, say, a fixed fractional method. This depends on the relationship between delta, the beginning account balance and the amount of profits made on consecutive winning trades, which is what can create immediate positive geometric growth. A caveat should accompany this—a series of winning trades that exceed delta early on can quickly increase the number of contracts traded, putting you at considerable risk if large drawdowns occur.

In the case of this event, the Fixed Ratio method will try to manage risk by adjusting trade size down. This is how the method deals with the issue of asymmetric leverage. It does not neglect drawdowns, but if the account does not lose more than the delta amount, the contract number will not be reduced, as it will have a chance to get

net positive on subsequent trades. This does not solve the problem of asymmetric leverage completely, but it is progress.

Handling drawdowns is a challenge for most position-size methods. There are variations that Jones talks about in his book that would help counter the effects of drawdown. If a trading strategy experiences low levels of drawdowns, a trader may want to investigate the Fixed Ratio method.

The second attribute addresses the risk-control function of the formula. As profits increase, the Fixed Ratio method begins to lower its position-size risk on each trade, which is the reverse of the way fixed fractional works. That is, the number of contracts traded decreases as a percentage of the account size as the account value increases. Jones says that users might want to employ a fixed fractional method at some point when the amount of risk taken on each trade gets very small relative to the profits in the account.

Jones' book includes a good example of how the delta value relates to the number of contracts traded and the account size. Below, Tables 3 and 4 exemplify a Fixed Ratio method with 10 futures contracts. In his book, Jones gives the calculation for the required account balance to add one contract as:

$$\begin{aligned} \text{Required Account Balance} = \\ \text{Previously Required Balance} + \\ (\text{Number of Contracts} * \text{Delta}) \end{aligned}$$

Tables 3 and 4 compare the differences between using a \$10,000 delta value to a \$1,000 delta value. The differences are dramatic. Looking at this example with a larger delta value of \$10,000, a much greater account size is needed to trade 10 contracts, or an account size of \$460,000. However, if the delta value is just \$1,000, the account size only needs to be a minimum of \$55,000 to trade 10 contracts. As mentioned earlier, with a lower delta value, we will be risking a greater percentage of our capital, which will mean that we will have greater leverage.

Table 3: Fixed ratio example (Delta = \$10,000)

Delta: \$ 10,000.00		
Number of contracts	Required account balance to add one contract	Percentage increase in account balance after adding one contract
1	\$ 10,000.00	
2	\$ 20,000.00	100.00%
3	\$ 40,000.00	100.00%
4	\$ 70,000.00	75.00%
5	\$ 110,000.00	57.10%
6	\$ 160,000.00	45.50%
7	\$ 220,000.00	37.50%
8	\$ 290,000.00	31.80%
9	\$ 370,000.00	27.60%
10	\$ 460,000.00	24.30%

Table 4: Fixed ratio example (Delta = \$1,000)

Delta: \$ 1,000.00		
Number of contracts	Required account balance to add one contract	Percentage increase in account balance after adding one contract
1	\$ 10,000.00	
2	\$ 11,000.00	10.00%
3	\$ 13,000.00	18.18%
4	\$ 16,000.00	23.08%
5	\$ 20,000.00	25.00%
6	\$ 25,000.00	25.00%
7	\$ 31,000.00	24.00%
8	\$ 38,000.00	22.58%
9	\$ 46,000.00	21.05%
10	\$ 55,000.00	19.57%

Fixed Ratio Strategy Performance

Again, the RSI strategy was used to evaluate the Fixed Ratio method. All of the same testing parameters that were used to test the Kelly Criterion were applied here. Our delta value was the same as the E-mini S&P 500 initial margin amount, or \$5,000. Figure 5 demonstrates the

equity curve for this money-management model. Notice the differences here in the shape of this equity curve versus half Kelly. It has a more linear positive slope than half Kelly. Also, note that it consistently continues to make new equity highs, except when it experienced a drawdown period around trades 50 to 100, just as the Kelly Criterion did. However, the Kelly method took longer to make new equity highs out of that drawdown.

Figure 4: Fixed ratio equity curve (delta = \$5,000)

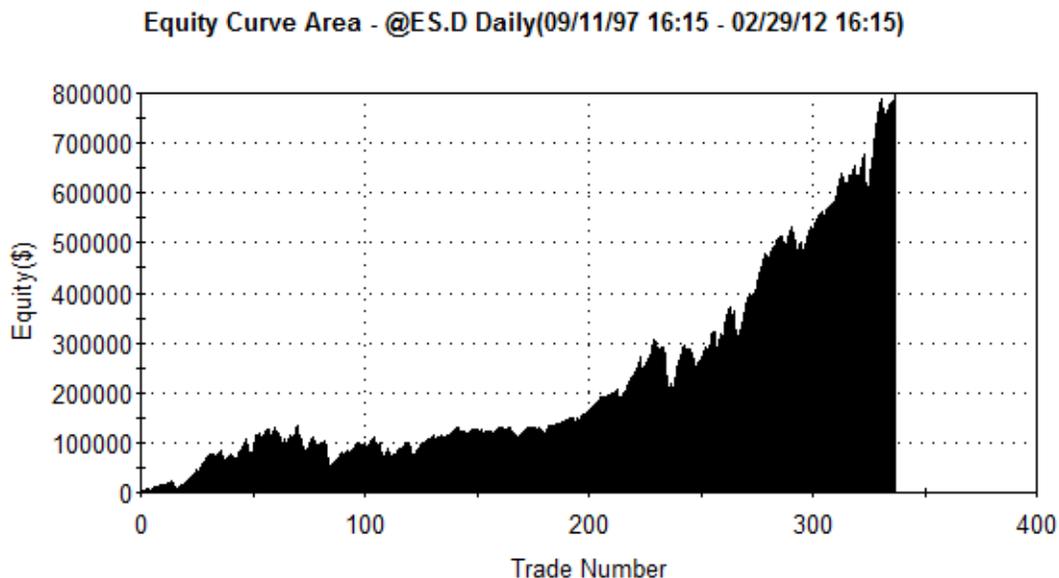
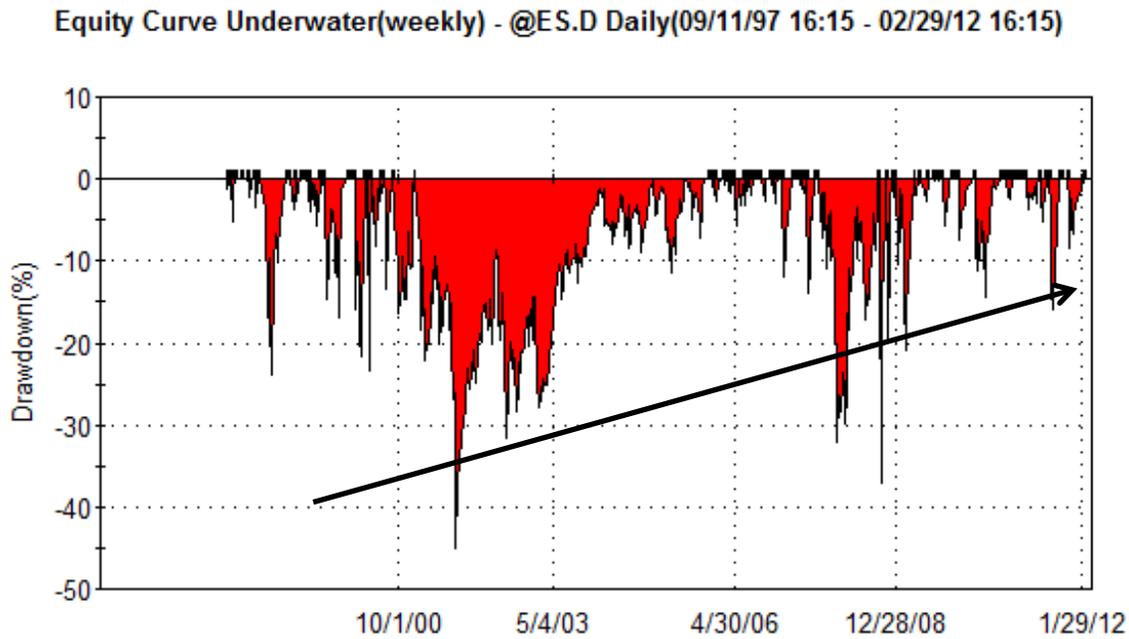


Figure 5: Fixed ratio underwater equity curve (Delta = \$5,000)



The fixed ratio does everything it was meant to do, as illustrated by the equity curve drawdowns in Figure 6. Notice how drawdowns increase in the beginning of the testing period, achieving their highest levels around the bottom of the 2000 - 2002 bear market. This type of performance can occur when the delta value is low, after a strategy has built up the number of contracts it trades from previous winning trades. Future periods of drawdown after the 45% levels that were met around September of 2001 are an attribute of this method that was noted previously. The drawdowns slope upward toward the current time period. As the account increases in value, the number of contracts at risk decreases, as a percentage of capital.

In Figure 7, the strategy performance statistics expose a higher profit factor of 1.60, compared to half Kelly's 1.54. Also, the return retracement ratio, Rina Index, Sharpe Ratio and K-Ratio all had greater values compared to half Kelly.

Figure 6: Fixed ratio strategy performance report statistics

TradeStation Performance Summary		
	All Trades	Long Trades
Total Net Profit	\$795,037.50	\$795,037.50
Gross Profit	\$2,122,062.50	\$2,122,062.50
Gross Loss	(\$1,327,025.00)	(\$1,327,025.00)
Profit Factor	1.60	1.60

Figure 7: Fixed ratio strategy performance report statistics

Return Retracement Ratio	0.21
RINA Index	85.64
Sharpe Ratio	0.20
K-Ratio	1.81

Conclusion

There is no perfect money-management model. As mentioned earlier, most money-management methods suffer from similar shortcomings. While risk is typically a key component in most of these models, many do a poor job of describing or incorporating risk directly in the model. When the risk component is not directly related to the capital losses a trader experiences, complacency about the degree of risk may result.

The central focus of money management is how to maximize the geometric return in a trading account, while risk control is secondary. As mentioned earlier, the trade-off dilemma is a critical issue. If you invest only a small portion of your account in each trade, you will not lose all your capital, but your account will grow very little over time. On the other hand, if you make the allocation to the trade large, the losses may cause you to suffer excessive drawdowns. So traders typically trade a fraction of what the method tells them should be allocated to the trade, as shown with the Kelly formula.

In conclusion, no money-management model can turn a trading system that has negative risk-adjusted expectancy into a positive risk-adjusted expectancy system. And despite what some say, a money-management model can turn a positive-expectancy system into a negative-expectancy system. It is imperative for the strategy trader to study the subject of money

management. These measures will be additive in preventing risk of ruin and finding the optimal trade-off between preferences toward risk and return.

The workspace and .eld files for the Fixed Ratio and Kelly Criterion money-management models are available for download with this paper.

Bibliography

1. Thorp, Edward O. *Beat the Dealer: A Winning Strategy for the Game of Twenty One*. New York: Random House, 1966.
2. J.L. Kelly, Jr. "A New Interpretation of Information Rate." *The Bell System Technical Journal*, 1956: 917-926.
3. Jones, Ryan. *The Trading Game: Playing by the Numbers to Make Millions*. New York: John Wiley & Sons, Inc., 1999.

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